Topic: Applications of Arithmetic and Geometric Sequences

Time: 45 mins Marks: /45 marks

Calculator Assumed

Question One: [2, 7, 2: 11 marks]

Elsa is negotiating with her mother as to how much pocket money she will get. Elsa suggests starting with $50 in the first month and increasing this by $5 every month.

a) With this scheme, how much pocket money will Elsa receive 12 months from the start?

Elsa's mother says that increasing the amount by 5% each month is better for Elsa in the long run.

b) i) Use the table below to show how much pocket money Elsa will receive with her scheme and her mother's scheme for the first 5 months of the year.

	Month 1	Month 2	Month 3	Month 4	Month 5
Elsa's scheme					
Mother's scheme					

ii) Is Elsa's mother correct? Justify your solution mathematically.

d) If the amount of pocket money Elsa will be paid is capped $120/month, does this effect which scheme is better? Justify your solution.

Question Two: [2, 5, 3: 10 marks]

Scientists are tracking the size of a colony of ants. Due to deaths, ants getting lost or leaving the colony the population decreases in size by 14.5% each year. 35 new ants are born per year and they always seem to be born at the end of the year.

There were 1200 ants in the colony when scientists first started tracking this colony. Shown below is a partially completed table.

Year	Ants at the start of the year	Number of deaths	Number of ants born	Population at the end of the year
1	1200	174	35	1061
2	1061	A	35	B
3	942.155 ~942	136.612 ~137	35	840.543 ~841
10	C	68.940 ~69	35	D

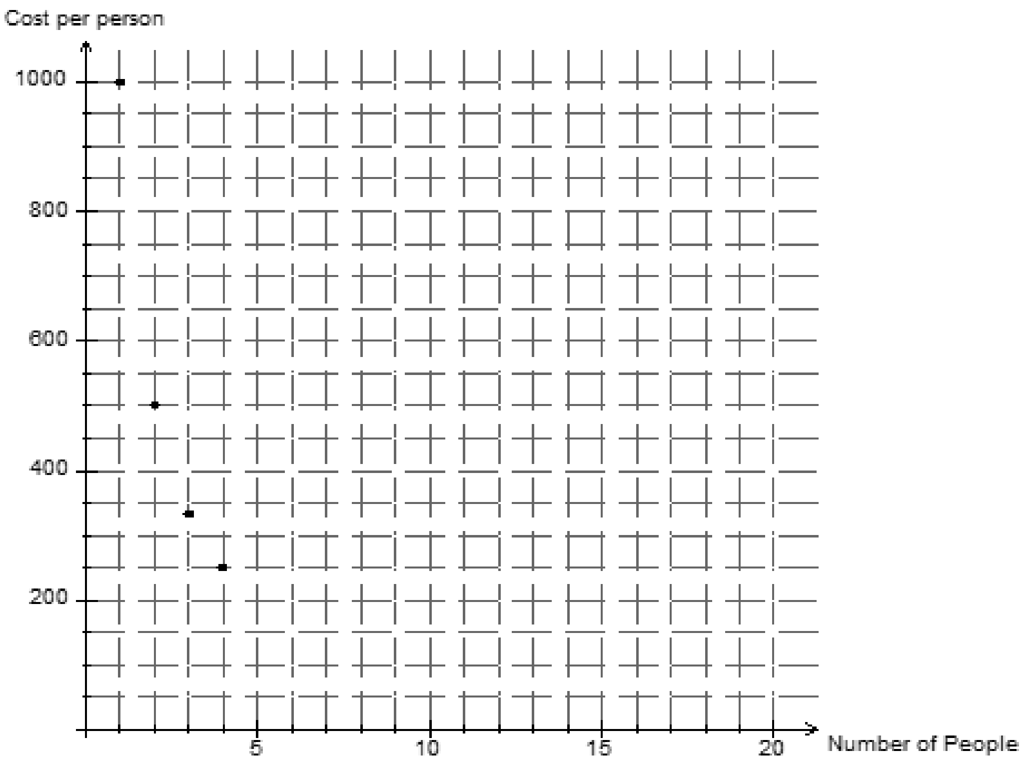
a) Calculate the values of A and B in the table above.

b) Write a recursive rule to determine the population of ants at the end of each year and hence or otherwise calculate the values of C and D.

c) When will the population of the colony remain constant/stable? Explain how this is mathematically possible and state the population at this time.

Question Three: [2, 2, 3, 2: 9 marks]

A group of friends want to hire a bus to take them away for a camping trip. They will split the cost of the hire per person. The following graph partially models the cost per person of hiring the bus depending the number of people included.



a) Add to the graph above the cost if the bus is hired by 5 people, 10 people and 20 people.

b) Is this relationship arithmetic, geometric or neither? Justify your reasoning.

Another bus company has a completely different model for hiring out the bus. They charge a $100 bond and then $30 per person.

c) Complete the following table showing the total cost of hiring a bus from the second company.

Number of People	1	2	3	20
Total cost				

d) Write a recursive rule which could be used to describe the total cost of the bus hire with the second company.

Question Four: [8, 7: 15 marks]

The following are the first three terms of a sequence: $-10m + 56, -3m + 4, m + 36, \frac{-m}{5} \ldots$

a) Assuming the sequence is geometric calculate the value of m and state the first four terms of the sequence.

b) If the first three terms of the sequence above form the start of an arithmetic sequence, calculate the value of m and state the first four terms of the sequence.

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Topic: Applications of Arithmetic and Geometric Sequences SOLUTIONS

Time: 45 mins ✓ Marks: /45 marks

Calculator Assumed

Question One: [2, 7, 2: 11 marks]

Elsa is negotiating with her mother as to how much pocket money she will get. Elsa suggests starting with \$50 in the first month and increasing this by \$5 every month.

a) With this scheme, how much pocket money will Elsa receive 12 months from the start?

$$T_{12} = 50 + 5(12 - 1) = \$105$$ ✓

In 12 months time her pocket money is \$105 for the month ✓

Elsa's mother says that increasing the amount by 5% each month is better for Elsa in the long run.

b) i) Use the table below to show how much pocket money Elsa will receive with her scheme and her mother's scheme for the first 5 months of the year.

	Month 1	Month 2	Month 3	Month 4	Month 5
Elsa's scheme	\$50	\$55	\$60	\$65	\$70
Mother's scheme	\$50	\$52.50	\$55.13	\$57.88	\$60.78

✓ ✓

ii) Is Elsa's mother correct? Justify your solution mathematically.

$T_{n+1} = T_n + 5, \quad T_1 = 50 \qquad T_{28} = \$185 \qquad T_{n+1} = T_n \times 1.05, \quad T_1 = 50 \qquad T_{28} = \$186.67$

✓ ✓ ✓ ✓

Elsa's mother's scheme is better in the long run. By the 28th month Elsa will be getting more pocket money under her mother's scheme and it increases exponentially. ✓

c) If the amount of pocket money Elsa will be paid is capped \$120/month, does this effect which scheme is better? Justify your solution.

Yes, in that case, Elsa's scheme is better because she reached \$120 first under her scheme.

✓ ✓

Question Two: [2, 5, 3: 10 marks]

Scientists are tracking the size of a colony of ants. Due to deaths, ants getting lost or leaving the colony the population decreases in size by 14.5% each year. 35 new ants are born per year and they always seem to be born at the end of the year.

There were 1200 ants in the colony when scientists first started tracking this colony. Shown below is a partially completed table.

Year	Ants at the start of the year	Number of deaths	Number of ants born	Population at the end of the year
1	1200	174	35	1061
2	1061	A	35	B
3	942.155 ~942	136.612 ~137	35	840.543 ~841
10	C	68.940 ~69	35	D

a) Calculate the values of A and B in the table above.

$A = 153.845 \sim 154$ $\quad B = 942.155 \sim 942$

b) Write a recursive rule to determine the population of ants at the end of each year and hence or otherwise calculate the values of C and D.

$T_{n+1} = T_n \times 0.855 + 35, \quad T_1 = 1061 \text{ or } \{A_0 = 1200\}$

$C = 475.447 \sim 475 \qquad D = 441.507 \sim 442$

c) When will the population of the colony remain constant/stable? Explain how this is mathematically possible and state the population at this time.

By the start of the 59^{th} year the population of ants is 241. 14.5% of 241 is 35. Meaning the number of ants who die is the same as the number of ants being born.

Question Three: [2, 2, 3, 2: 9 marks]

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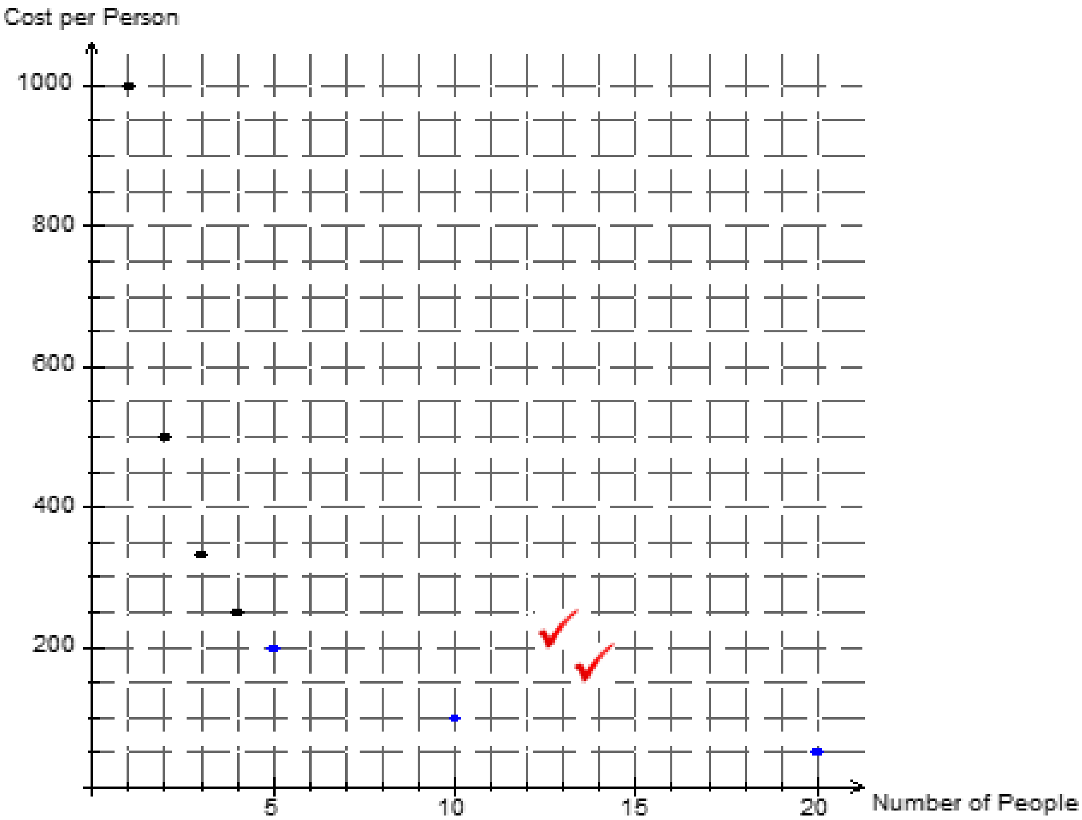

a) Add to the graph above the cost is the bus is hired by 5 people, 10 people and 20 people.

b) Is this relationship arithmetic, geometric or neither? Justify your reasoning.

Neither, the relationship is inverse proportion. There is no constant difference or constant multiplier so it is not an arithmetic or geometric relationship.

Another bus company has a completely different model for hiring out the bus. They charge a $100 bond and then $30 per person.

c) Complete the following table showing the total cost of hiring a bus from the second company.

Number of People	1	2	3	20
Total cost	$130	$160	$190	$700 ✓

✓ ✓

d) Write a recursive rule which could be used to describe the total cost of the bus hire with the second company.

$$T_{n+1} = T_n + 30, \qquad T_1 = 130\ \{T_0 = 100\}$$

✓ ✓

Question Four: [8, 7: 15 marks]

The following are the first three terms of a sequence: $-10m + 56, -3m + 4, m + 36, \frac{-m}{5} \ldots$

a) Assuming the sequence is geometric calculate the value of m and state the first four terms of the sequence.

$$\frac{-3m + 4}{-10m + 56,} = \frac{m + 36}{-3m + 4}$$

$$m = -20 \; m = 5.26$$

$$\frac{\frac{-m}{5}}{m + 36} = \frac{-3m + 4}{-10m + 56}$$

$$\textit{only solution is } m = -20$$

$$T_1 = 256 \qquad T_2 = 64 \qquad T_3 = 16 \qquad T_4 = 4$$

b) If the first three terms of the sequence above form the start of an arithmetic sequence, calculate the value of m and state the first four terms of the sequence.

$$-3m + 4 - (-10m + 56) = m + 36 - (-3m + 4)$$

$$m = 28$$

$$T_1 = -224 \quad T_2 = -80 \quad T_3 = 64 \quad T_4 = 64 + 144 = 208$$